

Quantum Zeno and Zeno-like effects in nitrogen vacancy centers

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ABSTRACT

We present a proposal to realize the quantum Zeno effect (QZE) and quantum Zeno-like effect (QZLE) in a proximal ^{13}C nuclear spin by controlling a proximal electron spin of a nitrogen vacancy (NV) center. The measurement is performed by applying a microwave pulse to induce the transition between different electronic spin states. Under the practical experimental conditions, our calculations show that there exist both QZE and QZLE in a ^{13}C nuclear spin in the vicinity of an NV center.

Quantum Zeno effect^{1,2} (QZE) is a very interesting phenomenon in quantum physics, in which the evolution of a quantum system can be inhibited by frequent measurements. In 1990, based on Cook's theoretical proposal,³ QZE⁴ was experimentally demonstrated by controlling the transition between two hyperfine levels of $^9\text{Be}^+$ with laser pulses, and there was a good agreement between theoretical prediction and experimental results. Since then, much effort has been paid to the research on QZE,⁵⁻⁸ quantum anti-Zeno effect⁹⁻¹² and quantum Zeno-like effect (QZLE).^{13,14} Because of its potential application in slowing down or even freezing the dynamic evolution of a system via repeated frequent measurements, it recently has attracted considerable interest as a tool in the fields of quantum information processing¹⁵⁻²¹ and ultrasensitive magnetometer.²²

QZE has been successfully demonstrated on various physical systems, such as trapped ions,²³ superconducting qubits,^{24,25} cavity quantum electrodynamics,^{7,16} nuclear magnetic resonance,^{26,27} and Bose-Einstein condensates.²⁸⁻³⁰ On the other hand, in solid-state quantum-information technology, a nitrogen vacancy (NV) center which consists of a nitrogen substituting for a carbon and an adjacent vacancy in diamond has been identified as one of the most promising candidates for qubits³¹⁻³⁸ due to its long coherence time at room temperature³⁹⁻⁴¹ and convenient manipulation under optical field, microwave field and rf field.^{42,43} Quantum Zeno-like phenomenon was experimentally demonstrated by inhibiting coherent spin dynamics induced by the microwave driving between two ground-state electron-spin levels ($m_s = 0$ and $m_s = 1$) of a single NV center.⁴⁴ Therein, only one measurement is performed to analyze the measurement effect on electron-spin states, i.e., the population variation between the electron-spin states $m_s = 0$ and $m_s = 1$ has been made by this single measurement.

In the conventional QZE, repeated instantaneous perfect measurements performed on the system will freeze the evolution of the initial state. The perfect conventional QZE requires infinite measurements with repetitive frequency approaching infinity, which might be impossible in experiment. However, it was recently discovered that perfect freezing of quantum states can also be achieved by more realistic non-projective measurements performed at a finite frequency.¹³ According to Brouwer's fixed-point theorem,⁴⁵ there always exist some quantum states which satisfy $\Phi(\rho_0) = \rho_0$, where Φ represents a quantum dynamical evolution process of a system with its initial state ρ_0 . After n identical cycle process, the system stays at the state as the same as its initial one, i.e., $\Phi^n(\rho_0) = \rho_0$. In this way, a QZLE can be achieved with finite-frequency measurements.

In this paper, inspired by the discovery in Ref. 13, we present a proposal to achieve the QZLE in a proximal ^{13}C nuclear spin of an NV center by controlling the electron spin. In our proposal, the electron spin plays the role as a detector while the ^{13}C nuclear spin acts as the target. Furthermore, the conventional QZE is demonstrated by modulating the measurement parameters and the external magnetic field. Here, instead of projective measurements, we apply a microwave pulse to induce the transition between different electronic states, followed by initialization of electron spin. Our numerical calculation properly shows that for suitable parameters there exist the conventional QZE and QZLE in a proximal ^{13}C nuclear spin of the NV center.

Results

The model

An NV center in diamond is composed of a nitrogen atom and a vacancy in an adjacent lattice site. It is a defect with C_{3v} symmetry.^{46,47} For the negatively-charged NV center with electron spin $S=1$, the ground state is a spin-triplet state 3A with a zero-field splitting $D=2.87$ GHz between spin sublevels $m_s = 0$ and $m_s = \pm 1$.⁴⁸ Around NV centers there are three kinds of nuclear spins,^{49,50} i.e., ^{13}C ($I=1/2$), ^{14}N ($I=1$), and ^{15}N ($I=1/2$). They can be manipulated by microwave and rf fields.

Consider an NV center and a ^{13}C nuclear spin which locates in the first coordination shell around the NV center,⁵¹ as shown in Fig. 1(a). In other words, this ^{13}C nuclear spin is at the nearest-neighbor lattice site of the NV center. As a result, there is a strong hyperfine coupling between the nuclear and electronic spins. Figure 1(b) shows the simplified energy-level diagram of the ground-state hyperfine structure associated with the nearest-neighbor ^{13}C nuclear spin. To demonstrate the QZE, the electron-spin states ($m_s = -1, 0$) and nuclear-spin states ($|\uparrow\rangle, |\downarrow\rangle$) are chosen to code qubits. The target and detector are initially uncorrelated, i.e., they are in a product state. A strong electron-spin polarization into the $m_s = 0$ sublevel can be induced by circulatory optical excitation-emission. This effect results from spin-selective non-radiative intersystem crossing to a metastable state lying between the ground and excited triplet states.^{52,53} Moreover, the nuclear spin could be well isolated from the electron spin, during the optical polarization and measurement of the electronic state.^{42,54} In other words, the state of nuclear spin could be unperturbed when the initialization and measurement are performed on the electronic spin.

Suppose that the electron spin is initially in its ground state $|0\rangle$ and the nuclear spin is in an arbitrary state. First of all, the whole system evolves freely for a time interval Δt_f . Afterwards, a microwave driving is used to perform measurement. As Fig. 1(b) shows, the microwave drives the transition between $|0, \uparrow\rangle$ and $|-1, \uparrow\rangle$ with Rabi frequency Ω and driving frequency ω . In the process of measurement, the total system evolves under the Hamiltonian $H_M = H_F + H_I$ for a time interval Δt_m , where H_F is the free Hamiltonian without measurement and H_I is the interaction Hamiltonian describing the transition induced by microwave driving. After the measurement, by optical pumping, the electron spin is initialized in its ground state $|0\rangle$, and meanwhile the electron and nuclear spins are decoupled.^{42,54,55} And then the above process is repeated. When the duration of the 532-nm light pulse for optical pumping is appropriate, the ^{13}C nuclear spin could be well isolated from the electron spin and the nuclear spin state can be preserved. In particular, the dephasing of nuclear spin can hardly be observed for light pulses of ~ 140 ns which is sufficiently long to polarize the electron spin while leave the state of nuclear spin undisturbed.⁴²

The free Hamiltonian H_F

The general Hamiltonian of an NV center and a ^{13}C nuclear spin which locates in the first coordination shell around the NV center is⁵⁶

$$H = DS_z^2 + \gamma_e \vec{B} \cdot \vec{S} + \gamma_n \vec{B} \cdot \vec{I} + \vec{S} A \vec{I} \quad (1)$$

$$= DS_z^2 + \gamma_e (B_x S_x + B_y S_y + B_z S_z) + \gamma_n (B_x I_x + B_y I_y + B_z I_z) + (A_{xx} S_x I_x + A_{yy} S_y I_y + A_{zz} S_z I_z).$$

Here, the first term stands for the zero-field splitting of the electronic ground state. The second term is the Zeeman energy splitting of the electron with γ_e being the electronic gyromagnetic ratio. The third term denotes the nuclear Zeeman effect where γ_n is the ^{13}C nuclear spin gyromagnetic ratio. And the last term describes the hyperfine interaction between the electron spin and the nuclear spin of ^{13}C atom.

Using a permanent magnet, an external magnetic field B_z is applied parallel to the NV axis. Hence, $\gamma_e (B_x S_x + B_y S_y)$ and $\gamma_n (B_x I_x + B_y I_y)$ are removed. Under the condition of weak magnetic field strength, the difference between the zero-field splitting $D = 2.87$ GHz and the electronic Zeeman splitting is much larger than the hyperfine interaction. In this situation, the electron-nuclear-spin flip-flop processes induced by the hyperfine interaction are sufficiently suppressed. Therefore, this allows for the secular approximation,^{56–58} and the $S_x I_x$ and $S_y I_y$ terms can be neglected. In other words, for the weak external magnetic field along the NV axis only the longitudinal hyperfine interaction needs to be taken into account, and the ground-state manifold of the NV center coupled with a proximal ^{13}C nuclear spin is described by the Hamiltonian

$$H_F = DS_z^2 + \gamma_e B_z S_z + \gamma_n B_z I_z + A_{zz} S_z I_z, \quad (2)$$

where

$$I_z = \frac{1}{2} \sigma_z, \quad (3)$$

$$S_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad (4)$$

and σ_z is the Pauli-z operator.

The Hamiltonian under measurement

A microwave driving is utilized to perform the measurement, as shown in Fig. 1(b). This microwave pulse drives the transition between $|0\rangle$ and $|-1\rangle$. The driving frequency is set to be resonant with the transition between $|0, \uparrow\rangle$ and $|-1, \uparrow\rangle$, and meanwhile largely detuned from that between $|0, \downarrow\rangle$ and $|-1, \downarrow\rangle$. In this way, the transition between $|0, \uparrow\rangle$ and $|-1, \uparrow\rangle$ can be induced selectively. Thus the system evolves under the whole Hamiltonian $H_M = H_F + H_I$, where the interaction Hamiltonian is

$$H_I = \Omega e^{i\omega t} |0, \uparrow\rangle \langle -1, \uparrow| + \text{h.c.} \quad (5)$$

In order to analytically calculate the quantum dynamics under the influence of H_M , we transform to the rotating frame defined by the transformation $|\Psi(t)^R\rangle = U^\dagger(t) |\Psi(t)\rangle$, where $U(t) = \exp(-iH_F t)$, $|\Psi(t)\rangle$ and $|\Psi(t)^R\rangle$ are the wave functions in the static and rotating frames respectively. Therefore, using equation (23), the Hamiltonian under the rotating frame can be obtained, i.e.,

$$H_M^R = U^\dagger H_M U - H_F = \Omega |0, \uparrow\rangle \langle -1, \uparrow| + \text{h.c.} \quad (6)$$

Dynamic Evolution

Now, let us demonstrate the quantum dynamics of the whole system in a full cycle, which includes a free evolution followed by a measurement process. The electron spin is initially prepared in its ground state, i.e.,

$$\rho^{(D)}(0) = |0\rangle \langle 0|, \quad (7)$$

and the nuclear spin is in an arbitrary state

$$\rho^{(T)}(0) = \begin{pmatrix} \alpha & \beta \\ \beta^* & 1 - \alpha \end{pmatrix}, \quad (8)$$

which is spanned by the basis $\{|\uparrow\rangle, |\downarrow\rangle\}$. Thus, the initial state of the total system is $\rho(0) = \rho^{(D)}(0) \otimes \rho^{(T)}(0)$.

In the free evolution, the total system evolves under its free Hamiltonian H_F for a time interval Δt_f , which is described by the evolution operator

$$U_f(\Delta t_f) = e^{-iH_F \Delta t_f}. \quad (9)$$

Apparently, without the driving, the evolution operator of the total system is diagonal. At the end of free evolution, the state of the total system becomes $\rho(\Delta t_f) = U_f(\Delta t_f) \rho(0) U_f^\dagger(\Delta t_f)$. Afterwards, a microwave pulse is used to drive the transition between $|0, \uparrow\rangle$ and $|-1, \uparrow\rangle$. The total system evolves under the Hamiltonian H_M^R for a time interval Δt_m . Having transformed to the rotating frame, a time-independent Hamiltonian is obtained and the corresponding evolution operator is

$$U_m^R(\Delta t_m) = e^{-iH_M^R \Delta t_m}. \quad (10)$$

After a cycle with duration $\tau = \Delta t_f + \Delta t_m$, the state of the whole system is

$$\rho^R(\tau) = U_m^R U_f \rho(0) U_f^\dagger U_m^{R\dagger}. \quad (11)$$

Utilizing equation (24), the final state of the whole system in the static frame reads

$$\rho(\tau) = U(\Delta t_m) U_m^R(\Delta t_m) U_f(\Delta t_f) \rho(0) U_f^\dagger(\Delta t_f) U_m^{R\dagger}(\Delta t_m) U^\dagger(\Delta t_m). \quad (12)$$

By partially tracing over the degree of the electron spin, the final state of the nuclear spin $\rho^{(T)}(\tau) = \text{Tr}_D[\rho(\tau)]$ reads

$$\rho^{(T)}(\tau) = \begin{pmatrix} \alpha & e^{-i\gamma_n B_z \tau} \cos(\Omega \Delta t_m) \beta \\ e^{i\gamma_n B_z \tau} \cos(\Omega \Delta t_m) \beta^* & 1 - \alpha \end{pmatrix}. \quad (13)$$

On the other hand, the initial state of the nuclear spin can be decomposed into its eigenbasis as¹³

$$\rho^{(T)}(0) = C_0 I + C_1 \sigma_+ + C_2 \sigma_- + C_3 \sigma_z, \quad (14)$$

where $C_0 = 1/2$, $C_1 = \beta$, $C_2 = \beta^*$, $C_3 = (2\alpha - 1)/2$, I is the identity operator, $\sigma_+ = |\uparrow\rangle \langle \downarrow|$ and $\sigma_- = |\downarrow\rangle \langle \uparrow|$ are the raising and lowering operators respectively. After the first cycle, the nuclear spin is in the state

$$\rho^{(T)}(\tau) = C_0 \lambda_0 I + C_1 \lambda_1 \sigma_+ + C_2 \lambda_2 \sigma_- + C_3 \lambda_3 \sigma_z, \quad (15)$$

where the eigenvalues are

$$\lambda_0 = \lambda_3 = 1, \quad (16)$$

$$\lambda_1 = \lambda_2^* = e^{-i\gamma_n B_z \tau} \cos(\Omega \Delta t_m). \quad (17)$$

After N cycles, the nuclear-spin qubit evolves into the state

$$\rho^{(T)}(N\tau) = C_0 \lambda_0^N I + C_1 \lambda_1^N \sigma_+ + C_2 \lambda_2^N \sigma_- + C_3 \lambda_3^N \sigma_z. \quad (18)$$

Here, $\lambda_0 = \lambda_3 = 1$ are related to fixed points⁴⁵ independent of all parameters. However, λ_1 and λ_2 are modulated by the parameters $(\Omega, B_z, \Delta t_m, \Delta t_f)$. By adjusting these parameters, the quantum Zeno and Zeno-like effects can be observed.

Hereafter, by analyzing the dependence of the eigenvalues on the parameters, we demonstrate the existence of quantum Zeno and Zeno-like effects.

Quantum Zeno-like effect

In equation (15), the eigenvalues $\lambda_0 = \lambda_3 = 1$ mean that $\rho^{(T)}(0) = C_0 I + C_3 \sigma_z$ are the fixed points independent of the combination of parameters $(\Omega, B_z, \Delta t_m, \Delta t_f)$ after repeated measurements. To be specific, if the initial state of the nuclear spin is of diagonal form, the state will not change and thus is preserved. This is a QZLE on the nuclear spin, similar to that in Ref. 13.

On the other hand, the σ_+ and σ_- components of the initial nuclear-spin state are exponentially suppressed, when $|\lambda_1| = |\lambda_2| < 1$ with appropriate parameters $(\Omega, B_z, \Delta t_m, \Delta t_f)$. Therefore, the following process $\rho^{(T)}(0) = C_0 I + C_1 \sigma_+ + C_2 \sigma_- + C_3 \sigma_z \rightarrow \rho^{(T)}(N\tau) = C_0 I + C_3 \sigma_z$ is achieved by sufficiently-many measurements.

Furthermore, the existence of $\lambda_3 = 1$ preserves the polarization of the nuclear spin. If α is 1 or 0, $\rho^{(T)}(N\tau) = \rho^{(T)}(0) = |\uparrow\rangle\langle\uparrow|$ or $\rho^{(T)}(N\tau) = \rho^{(T)}(0) = |\downarrow\rangle\langle\downarrow|$ can be obtained. Thus, the polarization of the ^{13}C nuclear spin near the NV center is frozen. It may be a potential way to preserve the polarization of ^{13}C nuclear spin against its hyperfine interaction with electron spin.

Quantum Zeno effect

For the eigenvalues λ_1 and λ_2 , we consider both the ideal situation when $\Delta t_f, \Delta t_m \rightarrow 0$ and the realistic situation of finite Δt_f and Δt_m .

In the ideal situation, equation (17) is simplified as

$$\lambda_1 = \lambda_2^* \approx e^{-i\gamma_n B_z \tau} \left(1 - \frac{\Omega^2}{2} \Delta t_m^2 \right). \quad (19)$$

From equation (18), we learn that the eigenvectors σ_+ and σ_- contribute to the off-diagonal part of $\rho^{(T)}(\tau)$. When Δt_f and Δt_m are small enough, the eigenvalues $|\lambda_1|$ and $|\lambda_2|$ can be very close to unity. Furthermore, $\gamma_n B_z N\tau = 2n\pi$ is assumed, where $N\tau = T$ is the fixed total evolution time and n is an integer. Because the eigenvalues $|\lambda_1|$ and $|\lambda_2|$ approach unity quadratically in the high-measurement-frequency limit, an arbitrary nuclear-spin state is exactly preserved by infinitely-frequent measurements. Here, the conventional QZE is recovered.

On the other hand, consider the realistic condition in an NV center, e.g. non-vanishing Δt_m due to a finite pulse width. When the Rabi frequency and the pulse width are chosen to meet the following requirement

$$\Omega \Delta t_m = 2n_1 \pi \quad (20)$$

with n_1 being positive integer, $\lambda_1 = \lambda_2 = 1$ can be obtained. In this case, arbitrary initial state is the Zeno-like fixed point which depend on the appropriate choice of parameters $(\Omega, B_z, \Delta t_m, \Delta t_f)$. In other words, under certain measurement conditions, the QZLE is observed. Figure 2 shows the locations where QZLE will occur in the parameter space of $(\Omega, \Delta t_m)$. In the case of $|\lambda_1| = |\lambda_2| < 1$, after repeated measurements the elements of σ_+ and σ_- in $\rho^{(T)}(T)$ may disappear due to accumulated loss. However, by tuning parameters to the vicinity of the points $(\Omega, \Delta t_m)$ given in equation (20), equation (17) can be expanded to the second order of Δt_m as

$$\lambda_1 = \lambda_2^* \approx 1 - \frac{1}{2}(\Omega \Delta t_m - 2n_1 \pi)^2. \quad (21)$$

Here, the conventional QZE happens in the neighbourhood of the QZLE points. In other words, the QZE occurs for a series of parameter combinations corresponding to finite-frequency measurements with finite coupling strengths. Since the eigenvalues λ_1 and λ_2 approach unity quadratically under the repeated measurements, any nuclear-spin state is exactly preserved by finitely-frequent measurements, even though it is affected by the free evolution.

Discussion

The conventional QZE and QZLE are demonstrated in a ^{13}C nuclear spin around an NV center by controlling the electron spin. Both of the QZE and QZLE can be observed by modulating the Rabi frequency, and the magnetic field, and the free-evolution time, and the pulse width. Our numerical calculation properly shows that for suitable parameters there exist both the QZE and QZLE in an NV-center system under the experimental condition. Consequently, the conventional QZE and QZLE are obtained with finite-frequency imperfect measurements.

In order to put our experimental proposal into practice, the secular approximation should be valid, i.e. the applied magnetic field strength should not be too strong.⁵⁶ As a consequence, the magnetic field strength B_z could be less than 200 G. Additionally, due to the resonance condition, the driving frequency ω equals to the level spacing between $|0, \uparrow\rangle$ and $|-1, \uparrow\rangle$, i.e., $D - \gamma_e B_z - A_{zz}/2$. At the same time, the level spacing ω_1 between $|0, \downarrow\rangle$ and $|-1, \downarrow\rangle$ is $D - \gamma_e B_z + A_{zz}/2$. To selectively only induce the transition between $|0, \uparrow\rangle$ and $|-1, \uparrow\rangle$, the large-detuning condition $\Delta\omega = \omega_1 - \omega = A_{zz} \gg \Omega$ should be fulfilled. Since the hyperfine coupling between the electron spin and a ^{13}C nuclear spin in the first coordination shell is known to be 130 MHz,^{51,59} the Rabi frequency Ω can be no more than 10 MHz. Furthermore, the initialization of the NV center will take approximately ~ 140 ns,⁴² and Δt_m and Δt_f are on a time scale about $2 \mu\text{s}$. Thus, a single cycle process will take about $5 \mu\text{s}$. The intrinsic dephasing time of the ^{13}C nuclear spin T_{2n} was observed as around one second.⁴¹ To ignore the decoherence effect induced by the environment, we restrict the total experiment time as $T \ll T_{2n}$, i.e., the total experiment time T is chosen as 100 ms. In this case, we can demonstrate the quantum Zeno and Zeno-like effects for roughly 2×10^4 cycles.

On the other hand, due to the presence of the nitrogen nucleus, ^{14}N ($I=1$) or ^{15}N ($I=1/2$), and the ^{13}C nuclear spin bath, the dephasing time of the electron spin is $58 \mu\text{s}$.³⁹ The duration $\tau = \Delta t_f + \Delta t_m$ of a cycle is smaller than the dephasing time of the electron spin by one order. Because the hyperfine coupling between the electron spin and the nitrogen nucleus $A_N < 4$ MHz⁵⁰ is much smaller than A_{zz} , the dephasing effect induced by the nitrogen nucleus can be neglected. The dipole-dipole interactions between the electron spin and the other ^{13}C nuclear spins are too weak. Therefore, the dephasing effect induced by all nuclear spins can also be neglected.

Last but not the least, the nuclear spin bath induces the dephasing of the ^{13}C nuclear spin. Because the total experiment time is sufficiently short and the magnetic dipole-dipole interactions between the ^{13}C nuclear spin and the other nuclear spins are weak enough,⁶⁰ the dephasing effect induced by the spin bath can be neglected. Meanwhile, after every measurement, we decouple the electron and ^{13}C nuclear spins and initialize the electron spin in its ground state $|0\rangle$ ^{42,54} without perturbing the ^{13}C nuclear spin. In this process, the nuclear spin is supposed to be completely isolated from the environment.^{42,54} Therefore, the nuclear spin hardly evolves during this process.

In conclusion, as shown in Fig. 2, our numerical calculation properly indicates that under practical conditions we can demonstrate the conventional QZE and QZLE in ^{13}C nuclear spin around the NV center with finite-frequency imperfect measurements.

Methods

The rotating frame. Since the original Hamiltonian in the measurement process H_M is time-dependent, the whole system is transformed to a rotating frame defined by the transformation $|\Psi(t)^R\rangle = U^\dagger(t)|\Psi(t)\rangle$, where $U(t) = \exp(-i\xi t)$, $|\Psi(t)\rangle$ and $|\Psi(t)^R\rangle$ are respectively the wave functions in the static and rotating frames. Now, we derive the relationship between the Hamiltonian in the static frame H_M and the Hamiltonian in the rotating frame H_M^R . Because the time evolution of $|\Psi(t)^R\rangle$ still fulfills the Schrodinger equation in the rotating frame, i.e.

$$\begin{aligned}
 i|\dot{\Psi(t)^R}\rangle &= i\frac{d}{dt}U^\dagger|\Psi(t)\rangle \\
 &= i\dot{U}^\dagger|\Psi(t)\rangle + U^\dagger i|\dot{\Psi(t)}\rangle \\
 &= i\dot{U}^\dagger U U^\dagger|\Psi(t)\rangle + U^\dagger H_M U U^\dagger|\Psi(t)\rangle \\
 &= i\dot{U}^\dagger U|\Psi(t)^R\rangle + U^\dagger H_M U|\Psi(t)^R\rangle \\
 &= (i\dot{U}^\dagger U + U^\dagger H_M U)|\Psi(t)^R\rangle \\
 &= (U^\dagger H_M U - \xi)|\Psi(t)^R\rangle,
 \end{aligned} \tag{22}$$

the effective Hamiltonian in the rotating frame reads

$$H_M^R = U^\dagger H_M U - \xi. \tag{23}$$

Correspondingly, the relationship between the density matrix in the static frame $\rho(t)$ and the density matrix in the rotating frame $\rho^R(t)$ is

$$\begin{aligned}\rho(t) &= |\Psi(t)\rangle\langle\Psi(t)| \\ &= U(t)|\Psi(t)^R\rangle\langle\Psi(t)^R|U^\dagger(t) \\ &= U(t)\rho^R(t)U^\dagger(t).\end{aligned}\tag{24}$$

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Author contributions statement

Q.A., Z.Q.Y., F.G.D., M.Z., J.Q. and Y.Y.W. wrote the main manuscript text, J.Q. and Y.Y.W. did the calculations. Q.A., F.G.D., M.Z. and Z.Q.Y. designed the project. Q.A. supervised the whole project. All authors reviewed the manuscript.

Additional information

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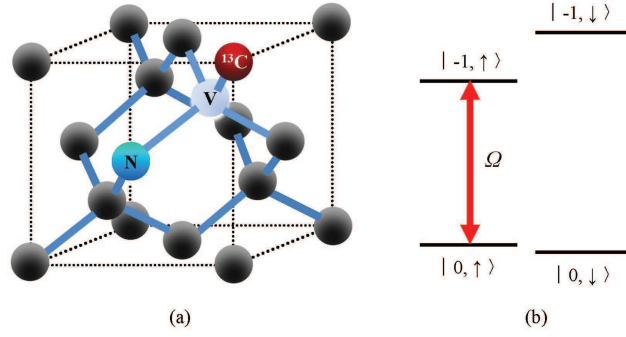


Figure 1. Scheme for demonstration of the QZE in an NV center. (a) A ^{13}C nuclear spin is at the nearest-neighbor lattice site of an NV center. (b) The energy-level diagram of the ground state hyperfine structure where a microwave drives the transition between $|0, \uparrow\rangle$ and $|-1, \uparrow\rangle$ with Rabi frequency Ω and driving frequency ω .

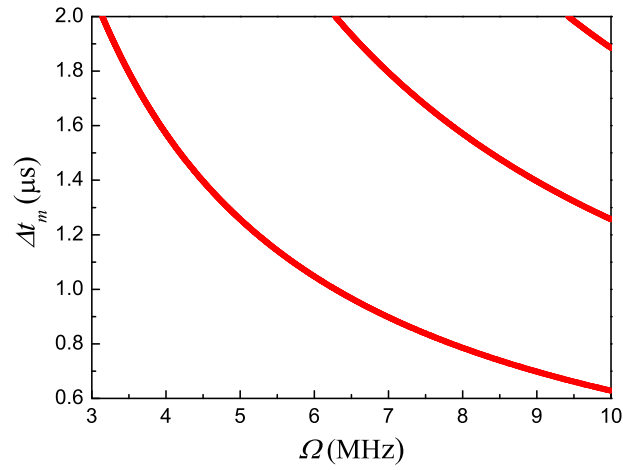


Figure 2. Measurement conditions for the QZLE. The QZLE will occur at the specified locations in the parameter space (Ω , Δt_m). The points are determined by equation (20) with $n_1 = 1, 2, 3$.